Multiplicative Sparse Modeling of Geometric Transformation Field

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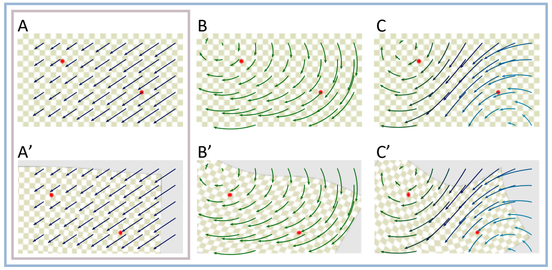
Overview

We developed Mathematical Analysis methods for Geometric Transformations by Substituting Additive operations with Multiplications in Their Formulations.

[Math Structure]
Elucidating Mathematical Structures in Real and Virtual World Objects and Their Utilization

Key aspects

Modeling deformation from few data by changing focus from "point-to-point" to "region-to-region".

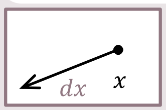


From the samples (red dots),

- Conventional vector fields capturing changes as "point-to-point" **cannot distinguish A – C.**
- By capturing changes as "region-to-region", it enables to represent **A – C differently**.

Movie

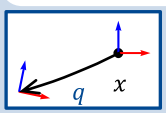
Vector Field: Displacement Vector is defined at point.



Vector
– only Translation
$x + dx$: Displacement Vector is Added.

Positions of red dots are identical in A – C.

Geometric Transform Field: Transform is defined at point.



Geometric Transformation (Rigid/Similarity)
– Translation + Rotation, Scaling
gx: Transform is Multiplied.

Surroundings of red dots are different among A – C.

Developed methods

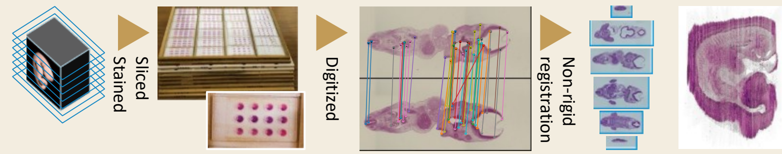
Sparse Regression | Graph Optimization | Spectral Decomposition

Results

We successfully reconstructed 3D shape of a human embryo from hundreds of serial sections produced more than 50 years ago.

Using Geometric Transformation Field to Model Deformations in Biological Tissues.

Target: Kyoto collection of Human Embryo
Hundreds of sections / Each section has **deformations** / **Difficult** to imagine **morphology in 3D**



▶ 3D Reconstruction from Serial section images

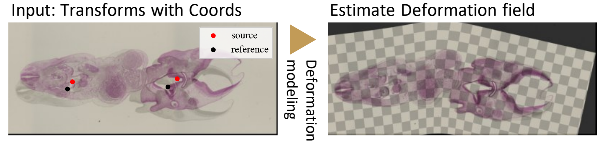
Challenges: Using sequential registration, accumulated error leads to large deformation.
Develop **Global optimization to suppress accumulated error.**

Sparse Regression
2D Rigid Transform

Result 1: Non-rigid Registration of Section images

Deformation field modeling from sparse keypoint matches

Input: Transforms with Coords — Estimate Deformation field



Graph Optimization
2D Rigid Transform

Result 2: 3D reconstruction from Serial sections

Global optimization performs non-rigid registration for hundreds of slices while minimizing deformation in each slice.

Input — Registration result

3D model

Spectral Decomposition

Result 3: Spectral Analysis

Decompose complex deformation in development process into simple deformation elements.

Applications

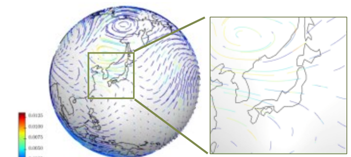
Wide variety of Application field from Metrological to Biomedical data analysis.

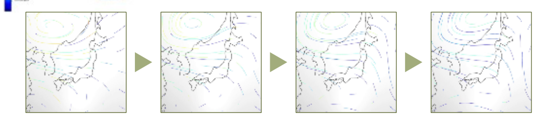
Sparse Regression
3D Rotation

App. 1: Cloud Advection modeling on Earth

Extracting sparse cloud motion from satellite images, then infer the advection in the globe.

3D model

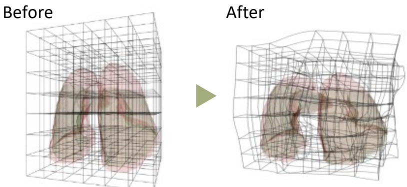




Sparse Regression
3D Similarity Transform

App. 2: 3D CT Registration

Models the deformation between inflation and exhalation.



3D model

Future work

- **Integration with DL techniques, other statistical analysis, and signal processing techniques.**
- **Exploring other applications.**